



FIG. 5. (a) Poisson's ratio of the rare-earth metals. (b) Bulk modulus of the rare-earth metals. Open points are estimated values.

directly related to Poisson's ratio, also shows more scatter than does either Young's modulus or the shear modulus.

**Boron, Ruthenium, and Rhenium.** Since no experimental value exists for Poisson's ratio for boron, ruthenium, and rhenium, this ratio was calculated from the experimental values of Young's and the bulk moduli. The values so obtained appear to be reasonable.

**Antimony.** No experimental value exists for Poisson's ratio for antimony, but from the experimental values of Young's, the shear, and the bulk moduli three different values for Poisson's ratio were calculated. By using Young's and the shear moduli, a value of 0.371 was obtained; by using Young's and the bulk moduli, 0.284 was obtained; and by using the shear and the bulk moduli, 0.277 was obtained. A mean value of  $0.31 \pm 0.06$  was thus chosen to represent Poisson's ratio for antimony. This situation clearly illustrates the difficulties involved when the elastic properties are not measured on the same sample.

**Selenium.** Since Poisson's ratio has not been experimentally determined for selenium, an attempt was made to calculate it by using the experimental values of Young's modulus and the bulk modulus and Eq. (II.2), which gave a value of  $-0.562$ . This negative value indicates that possibly Young's modulus or the bulk modulus or both are in error. This situation, and that as noted with antimony, clearly shows that elastic properties should be measured on the same specimen if one uses these data to calculate the remaining one or two elastic properties. The value for selenium given herein has been estimated (see below).

**Estimated Data.** As noted earlier, the ratio of Young's modulus to the shear modulus,  $Y/\mu$  (see Section 22 and Table XIX), is nearly a constant for all materials, the mean value being 2.60. By simple algebra Eq. (II.1) can be rewritten as

$$Y/\mu = 2(1 + \sigma). \quad (5.1)$$

This indicates that Poisson's ratio,  $\sigma$ , and the ratio  $Y/\mu$  are essentially the same quantity. However, since the data for most of the values of Poisson's ratio, Young's modulus, and the shear modulus have been measured independently and directly, the value of the ratio  $Y/\mu$  gives us an independent check on the measured value of Poisson's ratio. Thus the estimated values of Poisson's ratio given in Table III are the average values obtained by estimating both Poisson's ratio (see below) and the ratio  $Y/\mu$  (see Section 22).

It was mentioned above that the value of Poisson's ratio is dependent on the atomic structure (Fig. 4). Thus, instead of assuming a value of 0.301 for those elements for which Poisson's ratio is unknown, an average value for each group was assumed to be a better estimate. The mean value of Poisson's ratio of antimony and bismuth was assumed for phosphorus and arsenic; the mean value of yttrium and lanthanum, for scandium and actinium; the mean of rhombohedral sulfur and tellurium, for selenium and polonium; the mean of lithium, sodium, and potassium, for rubidium, cesium, and francium; the mean of calcium and barium, for strontium and radium; the mean of molybdenum and ruthenium, for technetium; the mean of barium and ytterbium, for europium<sup>13</sup>; the mean of ruthenium and iron, for osmium; the mean of thorium and uranium, for protactinium; and the value of neptunium was assumed to be identical with that of uranium. The values of the rare earths, promethium, thulium, and lutetium, were estimated from the straight line shown in Fig. 5a. Since these latter three values were averaged with the  $\sigma$  values calculated from the estimated  $Y/\mu$  values (Eq. (5.1)) to give the "best" estimated value, the open points for promethium, thulium, and lutetium do not lie on the straight line shown in Fig. 5a.